Unit 4 (Post AP Exam) Functions Involving Parameters, Vectors and Matrices

Content Area: **Mathematics**
Course(s):
Time Period: **May**
Length: **7 weeks**
Status: **Published**

Unit Overview:

In Unit 4, students explore function types that expand their understanding of the function concept. Parametric functions have multiple dependent variables' values paired with a single input variable or parameter. Modeling scenarios with parametric functions allows students to explore change in terms of components. This component-based understanding is important not only in calculus but in all fields of the natural and social sciences where we seek to understand one aspect of a phenomenon independent of other confounding aspects. Another major function type in this unit involves matrices mapping a set of input vectors to output vectors. The capacity to map large quantities of vectors instantaneously is the basis for vector-based computer graphics. While students may see their favorite video game character trip and fall or seemingly move closer or farther, matrices implement a rotation on a set of vectors or a dilation on a set of vectors. The power of matrices to map vectors is not limited to graphics but to any system that can be expressed in terms of components of vectors such as electrical systems, network connections, and regional population distribution changes over time. Vectors and matrices are also powerful tools of data science as they can be used to model aspects of complex scientific and social science phenomena.

Enduring Understandings:

- Equations of conic sections can be reorganized and analyzed to produce adequate graphs.
- Matrices allow for a streamlined method for solving systems
- Matrices have properties similar to, but not the same as those of real numbers
- Parametric equations allow an independent time (or angle) variable to dictate both x and y position.

Career Readiness, Life Literacies, & Key Skills:

WRK.9.2.12.CAP.4	Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
WRK.9.2.12.CAP.5	Assess and modify a personal plan to support current interests and post-secondary plans.

Essential Questions:

- How can we analyze the vertical and horizontal aspects of motion independently?

- How can we determine when the populations of species in an ecosystem will be relatively steady?
- How does high resolution computer- generated imaging achieve smooth and realistic motion on screen with so many pixels?

Standards/Indicators/Student Learning Objectives (SLOs):

MA.N-VM	Vector and Matrix Quantities
MA.N-VM.A	Represent and model with vector quantities.
MA.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\lvert\boldsymbol{v}\rvert$, $\lvert\lvert\boldsymbol{v}\rvert\rvert$, $\boldsymbol{v}$).
MA.N-VM.A.2	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.N-VM.B	Perform operations on vectors.
MA.N-VM.B.4	Add and subtract vectors.
MA.N-VM.B.4a	Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
MA.N-VM.B.4b	Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
MA.N-VM.B.4c	Understand vector subtraction $\boldsymbol{v} - w$ as $\boldsymbol{v} + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
MA.N-VM.B.5	Multiply a vector by a scalar.
MA.N-VM.B.5a	Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(\boldsymbol{v}_x, \boldsymbol{v}$ subscript $y) = (c\boldsymbol{v}_x, c\boldsymbol{v}$ subscript $y)$.
MA.N-VM.B.5b	Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\lvert\lvert c\boldsymbol{v}\rvert\rvert = \lvert c\rvert\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\lvert c\rvert\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
MA.N-VM.C	Perform operations on matrices and use matrices in applications.
MA.N-VM.C.6	Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
MA.N-VM.C.7	Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
MA.N-VM.C.8	Add, subtract, and multiply matrices of appropriate dimensions.
MA.N-VM.C.9	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
MA.N-VM.C.10	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
MA.N-VM.C.11	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
MA.N-VM.C.12	Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute

	value of the determinant in terms of area.
MA.A-REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.
MA.A-REI.C.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Career Education Connection

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Computer Science and Design Thinking Standards

CS.9-12.8.1.12.AP.5	Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.
CS.9-12.8.1.12.CS.2	Model interactions between application software, system software, and hardware.

Assessments

Summative Assessment:

- Alternate Assessment
- Benchmark
- Marking Period Assessment

Benchmark Assessments

Writing Prompt

Skills Based Assessment

Reading Response

Alternative Assessment

Performance tasks

Project-based assignments

Problem-based assignments

Presentations

Reflective pieces

Concept maps

Case-based scenarios

Portfolios

Formative Assessment:

- Anticipatory Set
- Closure
- Warm-Up

Lesson Titles:

- 4.1 Parametric Functions
- 4.10 Matrices
- 4.11 The Inverse and Determinant of a Matrix
- 4.12 Linear Transformations and Matrices
- 4.13 Matrices of Functions
- 4.14 Matrices Modeling Context
- 4.2 Parametric Functions Modeling Planar Motion
- 4.3 Parametric Functions and Rate of Change
- 4.4 Parametrically Defined Circles and Lines
- 4.5 Implicity Defined Functions
- 4.6 Conic Sections
- 4.7 Parametrization of Implicity Defined Functions
- 4.8 Vectors
- 4.9 Vector Valued Functions

Diversity, Equity, and Inclusion

Amistad Mandate

Topic:

Materials Used:

Addresses the Following Component of the Mandate:

- African Slave Trade
- Amistad
- Contributions of African Americans to our Society
- Slavery in America
- Vestiges of Slavery in this Country

Holocaust Mandate

Topic:

Materials Used:

Addresses the Following Component of the Mandate:

- Bias
- Bigotry
- Bullying
- Holocaust Studies
- Prejudice

LGBTQ and Disabilities Mandate

Topic (Person and Contribution Addresses):

Materials Used:

Addresses the Following Component of the Mandate:

- Economic
- Political
- Social

Climate Change

Asian American Pacific Islander Mandate

Topic (Person and Contribution Addresses):

Materials Used:

Addresses the Following Component of the Mandate:

- Economic
- Political
- Social

Inter-Disciplinary Connections:

9-12.HS-ETS1-4.5.1	Use mathematical models and/or computer simulations to predict the effects of a design solution on systems and/or the interactions between systems.
9-12.HS-ETS1-2.6.1	Design a solution to a complex real-world problem, based on scientific knowledge, student-generated sources of evidence, prioritized criteria, and tradeoff considerations.

Materials:

Core Instructional Materials

- AP Classroom

Supplemental Materials

- Calc Medic
- Collegeboard
- Flamingo Math
- Flipped Math

Texts at Various Levels

Instructional Strategies, Learning Activities, and Levels of Blooms/DOK:

- AP Classroom progress checks
- Define circle
- Define Determinant and inverse of matrix
- Define ellipse
- Define hyperbola
- Define matrix
- Define Parabola
- Define Parametric equation
- Define vector
- demonstrate matrices on calculator
- Demonstrate parametric functions modeling Planar motion
- Demonstrate standard form and general form for specific conic section
- Graph vectors
- Group activity
- Individual activity
- Intro Conic sections
- Practice solving Parametric functions
- Review completing the square
- thin slicing lesson
- vertical White board activity

Modifications

MLL Modifications:

- Choice of test format (multiple-choice, essay, true-false)
- Continue practicing vocabulary
- Provide study guides prior to tests
- Read directions to the student
- Read test passages aloud (for comprehension assessment)
- Vary test formats

G&T Modifications:

- Alternate assignments/enrichment assignments
- Enrichment projects
- Extension activities
- Higher-level cooperative learning activities
- Pairing direct instruction with coaching to promote self-directed learning
- Provide higher-order questioning and discussion opportunities
- Provide texts at a higher reading level
- Tiered assignments
- Tiered centers

At Risk Modifications

The possible list of modifications/accommodations identified for Special Education students can be utilized for At-Risk students. Teachers should utilize ongoing methods to provide instruction, assess student needs, and utilize modifications specific to the needs of individual students. In addition, the following may be considered:

- Additional time for assignments
- Adjusted assignment timelines
- Agenda book and checklists
- Answers to be dictated
- Assistance in maintaining uncluttered space
- Books on tape
- Concrete examples
- Extra visual and verbal cues and prompts
- Follow a routine/schedule
- Graphic organizers
- Have students restate information
- No penalty for spelling errors or sloppy handwriting
- Peer or scribe note-taking
- Personalized examples

- Preferential seating
- Provision of notes or outlines
- Reduction of distractions
- Review of directions
- Review sessions
- Space for movement or breaks
- Support auditory presentations with visuals
- Teach time management skills
- Use of a study carrel
- Use of mnemonics
- Varied reinforcement procedures
- Work in progress check

IEP & 504 Modifications:

*All teachers of students with special needs must review each student's IEP. Teachers must then select the appropriate modifications and/or accommodations necessary to enable the student to appropriately progress in the general curriculum.

Possible Modifications/Accommodations: (See listed items below):

- Allow for redos/retakes
- Assign fewer problems at one time (e.g., assign only odds or evens)
- Differentiated center-based small group instruction
- Extra time on assessments
- Highlight key directions
- If a manipulative is used during instruction, allow its use on a test
- Opportunities for cooperative partner work
- Provide reteach pages if necessary
- Provide several ways to solve a problem if possible
- Provide visual aids and anchor charts
- Test in alternative site
- Tiered lessons and assignments
- Use of a graphic organizer
- Use of concrete materials and objects (manipulatives)
- Use of word processor

Technology Materials and Standards

- AP Classroom Site
- Calc Medic Videos

- Flipped Math videos
- Google Classroom
- Google Slides
- Graphing Calculator
- IXL

TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.
TECH.8.1.12.F.CS4	Use multiple processes and diverse perspectives to explore alternative solutions.