

Unit #08: Exponential and Logarithmic Functions

Content Area: **Mathematics**
Course(s): **English I, Algebra II**
Time Period: **May**
Length: **4 weeks**
Status: **Published**

Unit Overview

This unit defines exponential function. Composites and inverses of functions are discussed completely, leading up to the definition of a logarithmic function as the inverse of exponential function. The laws of logarithms are given and applications are discussed.

Enduring Understandings

Students will understand how to use exponential logarithmic and natural logarithmic functions to solve real-world problems.

Students will understand there is more than one way to solve an exponential function.

Exponential functions and graphs display varying rates of change.

Essential Questions

- How might you use exponential, logarithmic, and natural logarithmic functions to solve real-world problems?
- When do quantities have a nonlinear relationship?
- Where can you find Exponential Decay in the real world?
- Where can you find Exponential Growth in the real world?

Lesson Titles/Objectives

- Define and use the natural logarithm function.
- Define logarithmic functions.
- Discover how logarithmic and exponential functions are related.
- Extend the meaning of exponents to include irrational numbers and to define exponential functions.
- Extend the meaning of exponents to include rational numbers.
- Find the composite of two given functions.
- Find the inverse of a given function.
- Understand and apply the basic properties of logarithms.

- Use common logarithms in problem solving.
- Use exponential and logarithmic functions to solve growth and decay problems.

Standards

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.6	Attend to precision.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Indicators

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly

using different properties of operations and objects.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.

Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-BF.A.1c	Compose functions.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.4	Find inverse functions.

21st Century Skills and Career Ready Practices

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Inter-Disciplinary Connections

LA.RL.11-12.1	Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicitly as well as inferences drawn from the text, including
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	determining where the text leaves matters uncertain.
LA.RL.11-12.4	Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including words with multiple meanings or language that is particularly fresh, engaging, or beautiful. (e.g., Shakespeare as well as other authors.)
LA.RI.11-12.10a	By the end of grade 11, read and comprehend literary nonfiction at grade level text-complexity or above with scaffolding as needed.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.2.E	Establish and maintain a style and tone appropriate to the audience and purpose (e.g., formal and objective for academic writing) while attending to the norms and conventions of the discipline in which they are writing.
LA.W.11-12.2.F	Provide a concluding paragraph or section that supports the argument presented (e.g., articulating implications or the significance of the topic).
LA.SL.11-12.1	Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively.
LA.SL.11-12.1.C	Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
LA.L.11-12.6	Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.
12.9.3.ST-SM.2	Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.
STEM.9-12.9.4.12.O.11	Apply active listening skills to obtain and clarify information.
STEM.9-12.9.4.12.O.12	Develop and interpret tables, charts, and figures to support written and oral communications.
STEM.9-12.9.4.12.O.17	Employ critical thinking skills (e.g., analyze, synthesize, and evaluate) independently and in teams to solve problems and make decisions.
STEM.9-12.9.4.12.O.22	Employ technological tools to expedite workflow.
STEM.9-12.9.4.12.O.48	Employ teamwork skills to achieve collective goals and use team members’ talents effectively.
STEM.9-12.9.4.12.O.55	Identify and demonstrate positive work behaviors and personal qualities needed to succeed.

Warm-Up

- Kahott!
- Sample PARCC item
- Sample SAT question
- Socratic

Anticipatory Set

- Pay it Forward video clip for exponential growth
- Use real life examples of exponential growth and decay (calling tree)

Instructional Strategies/Learning Activities

- Intro lesson on composition of functions and inverse of functions
- Intro lesson on defining a logarithm
- Intro lesson on exponential growth and decay
- Intro lesson on rational exponents
- Intro lesson on real number exponents
- Intro lesson on the laws of logarithms
- Intro lesson on the natural logarithm function
- Notes will be taken using Power Point
- Review homework
- Review warm up
- Students will present solutions on the board
- Students will work independently on examples
- Students will work together on a worksheet

Closure

- Discussion on today's lesson
- Exit ticket
- Journal Entry
- Kahoot!
- Oral Questioning
- Poll the class to self-analyze their comfort level of the lesson
- Socratic
- Vocab Review
- What did you learn today?

Modifications-G&T, LES, Special Education

- Collaborate with after-school programs or clubs to extend learning opportunities.
- Engage students with a variety of Mathematical Practices to provide students with multiple entry points and multiple ways to demonstrate their understandings.
- Provide ELL students with multiple literacy strategies.
- Provide multiple grouping opportunities for students to share their ideas and to encourage work among various backgrounds and cultures (e.g. multiple representation and multimodal experiences).
- Provide opportunities for students to connect with people of similar backgrounds (e.g. conversations

via digital tool such as SKYPE, experts from the community helping with a project, journal articles, and biographies).

- Provide students with multiple choices for how they can represent their understandings (e.g. multisensory techniques-auditory/visual aids; pictures, illustrations, graphs, charts, data tables, multimedia, modeling).
- Restructure lesson using UDL principals (http://www.cast.org/our-work/about-udl.html#.VXmoXcfD_UA)
- Structure lessons around questions that are authentic, relate to students' interests, social/family background and knowledge of their community.
- Structure the learning around explaining or solving a social or community-based issue.
- Use project-based math learning to connect math with observable phenomena.

Formative Assessment

- Applications of Logs Quiz
- Composition and Inverse of functions Quiz
- Group Work
- Guided Practice
- Individual Practice
- Kahoot!
- Natural Log Quiz
- Observation
- Oral Responses
- Rational and Real Number Exponents Quiz
- Smart Response
- Socrative
- Teacher Observation

Summative Assessment

- Unit Test on Exponential and Logarithmic Functions

Resources & Technology

- chromebook
- Desmos online graphing calculator
- Graphing Calculator
- Kahoot!
- mathxlforschool.com
- PowerPoint
- Smart Board

- Socrative.com
- Teacher generated worksheets
- Textbook: Algebra and Trigonometry Structure and Method Book 2 (McDougal Littell)
- Video to introduce or demonstrate concepts