

Unit #08: Exponential and Logarithmic Functions

Content Area: **Mathematics**
Course(s): **Algebra II**
Time Period: **May**
Length: **3 weeks**
Status: **Published**

Unit Overview

This unit defines exponential function. Composites and inverses of functions are discussed completely, leading up to the definition of a logarithmic function as the inverse of exponential function. The laws of logarithms are given and applications are discussed.

Enduring Understandings

Students will understand how to use exponential logarithmic and natural logarithmic functions to solve real-world problems.

Students will understand there is more than one way to solve an exponential function.

Exponential functions and graphs display varying rates of change.

Essential Questions

- How might you use exponential, logarithmic, and natural logarithmic functions to solve real-world problems?
- When do quantities have a nonlinear relationship?
- Where can you find Exponential Decay in the real world?
- Where can you find Exponential Growth in the real world?

Standards/Indicators/Student Learning Objectives (SLOs)

Student Learning Objectives:

- SWBAT define and use the natural logarithm function.
- SWBAT define logarithmic functions.
- SWBAT discover how logarithmic and exponential functions are related.
- SWBAT extend the meaning of exponents to include irrational numbers and to define exponential functions.
- SWBAT extend the meaning of exponents to include rational numbers.
- SWBAT find the composite of two given functions.

- SWBAT find the inverse of a given function.
- SWBAT understand and apply the basic properties of logarithms.
- SWBAT use common logarithms in problem solving.
- SWBAT use exponential and logarithmic functions to solve growth and decay problems.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.K-12.6	Attend to precision.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Indicators

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-BF.A.1c	Compose functions.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.4	Find inverse functions.

Lesson Titles

- Composite Functions
- Exponential Functions
- Exponential Growth and Decay
- Inverse Functions
- Natural Logarithm Function
- Problem Solving With Logarithms
- Properties of Logarithms

Career Readiness, Life Literacies & Key Skills

TECH.9.4.2.Cl.1	Demonstrate openness to new ideas and perspectives (e.g., 1.1.2.CR1a, 2.1.2.EH.1, 6.1.2.CivicsCM.2).
TECH.9.4.2.Cl.2	Demonstrate originality and inventiveness in work (e.g., 1.3A.2CR1a).

TECH.9.4.2.CT.2	Identify possible approaches and resources to execute a plan (e.g., 1.2.2.CR1b, 8.2.2.ED.3).
TECH.9.4.2.CT.3	Use a variety of types of thinking to solve problems (e.g., inductive, deductive).
TECH.9.4.2.DC.2	Explain the importance of respecting digital content of others.
TECH.9.4.2.TL.2	Create a document using a word processing application.
TECH.9.4.2.TL.3	Enter information into a spreadsheet and sort the information.

Inter-Disciplinary Connections

LA.RL.11-12.1	Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
LA.RL.11-12.4	Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including words with multiple meanings or language that is particularly fresh, engaging, or beautiful. (e.g., Shakespeare as well as other authors.)
LA.RST.9-10.1	Accurately cite strong and thorough evidence from the text to support analysis of science and technical texts, attending to precise details for explanations or descriptions.
LA.RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
LA.RI.11-12.10a	By the end of grade 11, read and comprehend literary nonfiction at grade level text-complexity or above with scaffolding as needed.
LA.WHST.9-10.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant sufficient textual and non-textual evidence.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.2.E	Establish and maintain a style and tone appropriate to the audience and purpose (e.g., formal and objective for academic writing) while attending to the norms and conventions of the discipline in which they are writing.
LA.W.11-12.2.F	Provide a concluding paragraph or section that supports the argument presented (e.g., articulating implications or the significance of the topic).
LA.SL.11-12.1	Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
LA.SL.11-12.1.C	Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
LA.L.11-12.6	Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.
12.9.3.ST-SM.2	Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.
STEM.9-12.9.4.12.O.11	Apply active listening skills to obtain and clarify information.
STEM.9-12.9.4.12.O.12	Develop and interpret tables, charts, and figures to support written and oral communications.
STEM.9-12.9.4.12.O.17	Employ critical thinking skills (e.g., analyze, synthesize, and evaluate) independently and in

	teams to solve problems and make decisions.
STEM.9-12.9.4.12.O.22	Employ technological tools to expedite workflow.
STEM.9-12.9.4.12.O.48	Employ teamwork skills to achieve collective goals and use team members' talents effectively.
STEM.9-12.9.4.12.O.55	Identify and demonstrate positive work behaviors and personal qualities needed to succeed.

Instructional Strategies, Learning Activities, and Levels of Blooms/DOK

- Bloom's Analysis: Break the concept into parts to understand how each is related to one another
- Bloom's Application: Use knowledge gained in new ways
- Bloom's Comprehension: Make sense of what has been learned
- Bloom's Evaluation: Put new information together in an innovative way
- Bloom's Knowledge: Recall relevant knowledge from prior lessons and long-term memory
- Bloom's Synthesis: Make judgements based on a set of guidelines to create new meaning
- Intro lesson on composition of functions and inverse of functions
- Intro lesson on defining a logarithm
- Intro lesson on exponential growth and decay
- Intro lesson on rational exponents
- Intro lesson on real number exponents
- Intro lesson on the laws of logarithms
- Intro lesson on the natural logarithm function
- Notes will be taken using Smart Notebook
- Review homework
- Review warm up
- Students will present solutions on the board
- Students will work independently on examples
- Students will work together on a worksheet
- Tutoring during Delsea One

Modifications

ELL Modifications

- Focus on domain specific vocabulary and keywords
- Offer alternate/or modify assessments
- Offer resources for specific topics in primary language (Youtube web resources)
- Provide formal and informal verbal interaction to provide practice, increase motivation, and self-

monitoring

- Tutoring during Delsea One

IEP & 504 Modifications

- Allow student to correct mistakes or answer wrong questions correctly for additional credit if failed the first test (another way to re-teach material)
- Allow student to take notes in class for reinforcement but also provide a copy of completed/correct notes to study from
- Model and show lots of examples
- Reduce homework length to just those most important for review
- Tutoring during Delsea One

G & T Modifications

- Ask students' higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning
- Determine where students' interests lie and capitalize on their inquisitiveness. (Is there a specific career they are interested in? How would this apply to their interest?)
- Encourage students to explore concepts in depth and encourage independent studies or investigations
- Provide additional rigorous challenge problems for advanced students

At Risk Modifications

- retesting
- review, restate, reword directions
- speaking to students privately when redirecting behaviors
- tutoring during Delsea One

Formative Assessment

- Exit Ticket
- Group Work
- Guided Practice
- Individual Practice
- Journal Entry
- Kahoot!
- Observation
- Oral Responses
- Poll class to self-analyze their comfort level of the lesson

- Socratic
- Teacher Observation
- Vocabulary Review

Alternate Assessment

Performance tasks

Project-based assignments

Problem-based assignments

Presentations

Benchmark Assessment

Skills-based assessment- math practice

Summative Assessment

- Alternative Assessment
- Marking Period Assessment
- Quiz on Applications of Logs
- Quiz on Composition and Inverse of Functions
- Quiz on Natural Log
- Quiz on Rational and Real Number Exponents
- Unit Test on Exponential and Logarithmic Functions

Resources & Materials

- Chromebook
- Graphing Calculator
- Promethean Board
- Smart Notebook
- Teacher generated worksheets
- Textbooks: Algebra and Trigonometry Structure and Method Book 2 (McDougal Littell), Algebra II Common Core (Pearson)

Technology

- google classroom
- <http://kutasoftware.com>
- http://mathxlforschool.com/home_school.htm
- <https://create.kahoot.it>
- <https://njctl.org/>
- <https://quizizz.com/>
- <https://socrative.com/>
- <https://www.desmos.com/>
- <https://www.resourceaholic.com/>
- Student 1-1 Device (chromebook)
- TI Graphing Calculator

TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
TECH.8.1.12.C.CS4	Contribute to project teams to produce original works or solve problems.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.