Unit #06: Irrational and Complex Numbers

Content Area: **Mathematics**
Course(s): **Algebra II**
Time Period: **March**
Length: **5 weeks**
Status: **Published**

Unit Overview

In this unit, students are introduced to roots and principal roots of real numbers. Students learn to use various properties to simplify, add, subtract, multiply, and divide radical expressions. The theme of solving equations is continued in this chapter by consideration of radical equations. This unit concludes with a study of roots of negative numbers and the arithmetic of complex numbers.

Enduring Understandings

Students will understand negatives under an even indexed radical are not part of the real number system and that imaginary numbers were invented to combat the natural phenomenon of a negative under a radical.

Students will understand solutions to quadratic equations include real and imaginary numbers.

Students will understand that real and imaginary numbers comprise the complex number system.

Essential Questions

- How do you distinguish between rational and irrational numbers?
- How might you work with complex numbers?
- Why would you have to use imaginary numbers?

Standards/Indicators/Student Learning Objectives (SLOs)

Student Learning Objectives:

- SWBAT develop the cycle of i.
- SWBAT distinguish the difference between rational and irrational numbers.
- SWBAT find roots of real numbers.
- SWBAT perform operations on complex numbers.
- SWBAT simplify expressions involving radicals.
- SWBAT simplify expressions involving sums of radicals.
- SWBAT simplify products and quotients of binomials that contain radicals.
- SWBAT solve equations containing radicals.

- SWBAT use the imaginary number i to simplify square roots of negative numbers.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.N-RN.B	Use properties of rational and irrational numbers.
MA.K-12.6	Attend to precision.
MA.F-IF.C	Analyze functions using different representations
MA.N-CN	The Complex Number System
MA.N-CN.A	Perform arithmetic operations with complex numbers.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.N-CN.A.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
MA.N-CN.A.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
MA.N-CN.A.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
MA.N-CN.B	Represent complex numbers and their operations on the complex plane.
MA.N-CN.B.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
MA.N-CN.B.5	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

Indicators

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of

their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y-2)/(x-1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1)$, $(x-1)(x^2+x+1)$, and $(x-1)(x^3+x^2+x+1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Lesson Titles

- Binomials with Radicals
- Operations on Complex Numbers
- Radical Equations
- Radical Operations

- Roots of Real Numbers
- Simplifying Radicals
- The Cycle of i
- The Imaginary Number i

Career Readiness, Life Literacies & Key Skills

TECH.9.4.2.CI.1	Demonstrate openness to new ideas and perspectives (e.g., 1.1.2.CR1a, 2.1.2.EH.1, 6.1.2.CivicsCM.2).
TECH.9.4.2.CI.2	Demonstrate originality and inventiveness in work (e.g., 1.3A.2CR1a).
TECH.9.4.2.CT.2	Identify possible approaches and resources to execute a plan (e.g., 1.2.2.CR1b, 8.2.2.ED.3).
TECH.9.4.2.CT.3	Use a variety of types of thinking to solve problems (e.g., inductive, deductive).
TECH.9.4.2.DC.1	Explain differences between ownership and sharing of information.
TECH.9.4.2.DC.3	Explain how to be safe online and follow safe practices when using the internet (e.g., 8.1.2.NI.3, 8.1.2.NI.4).
TECH.9.4.2.TL.2	Create a document using a word processing application.
TECH.9.4.2.TL.3	Enter information into a spreadsheet and sort the information.

Inter-Disciplinary Connections

LA.RST.9-10.2	Determine the central ideas, themes, or conclusions of a text; trace the text's explanation or depiction of a complex process, phenomenon, or concept; provide an accurate summary of the text.
LA.RST.9-10.3	Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
LA.RST.9-10.10	By the end of grade 10, read and comprehend science/technical texts in the grades 9-10 text complexity band independently and proficiently.
LA.RI.11-12.10a	By the end of grade 11, read and comprehend literary nonfiction at grade level text-complexity or above with scaffolding as needed.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.W.11-12.1.A	Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences claim(s), counterclaims, reasons, and evidence.
LA.W.11-12.2	Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
LA.WHST.9-10.9	Draw evidence from informational texts to support analysis, reflection, and research.
LA.L.11-12.6	Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.
12.9.3.ST-SM.1	Apply science and mathematics to provide results, answers and algorithms for engineering and technological activities.

12.9.3.ST-SM.2	Apply science and mathematics concepts to the development of plans, processes and projects that address real world problems.
STEM.9-12.9.4.12.O.2	Demonstrate mathematics knowledge and skills required to pursue the full range of postsecondary education and career opportunities.
STEM.9-12.9.4.12.O.4	Select and employ appropriate reading and communication strategies to learn and use technical concepts and vocabulary in practice.
STEM.9-12.9.4.12.O.47	Employ organizational skills to foster positive working relationships and accomplish organizational goals.
STEM.9-12.9.4.12.O.48	Employ teamwork skills to achieve collective goals and use team members' talents effectively.
STEM.9-12.9.4.12.O.51	Employ mentoring skills to assist others.

Instructional Strategies, Learning Activities, and Levels of Blooms/DOK

- Bloom's Analysis: Break the concept into parts to understand how each is related to one another
- Bloom's Application: Use knowledge gained in new ways
- Bloom's Comprehension: Make sense of what has been learned
- Bloom's Evaluation: Put new information together in an innovative way
- Bloom's Knowledge: Recall relevant knowledge from prior lessons and long-term memory
- Bloom's Synthesis: Make judgements based on a set of guidelines to create new meaning
- Discuss the term conjugate
- Intro lesson on adding and subtracting complex numbers
- Intro lesson on multiplying and dividing binomials with radicals
- Intro lesson on multiplying and dividing complex numbers
- Intro lesson on properties of radicals
- Intro lesson on roots of real numbers: square root, cube root, 4th root, etc
- Intro lesson on solving equations containing radicals
- Intro lesson on sums of radicals
- Intro lesson on the imaginary number i
- Notes will be taken using Smart Notebook
- Review homework
- Review warm up
- Students will present solutions on the board
- Students will work independently on examples
- Students will work together on a worksheet
- Tutoring during Delsea One

Modifications

ELL Modifications

- Focus on domain specific vocabulary and keywords
- Offer alternate/or modify assessments
- Offer resources for specific topics in primary language (Youtube web resources)
- Provide formal and informal verbal interaction to provide practice, increase motivation, and self-monitoring
- Tutoring during Delsea One

IEP & 504 Modifications

- Allow student to correct mistakes or answer wrong questions correctly for additional credit if failed the first test (another way to re-teach material)
- Allow student to take notes in class for reinforcement but also provide a copy of completed/correct notes to study from
- Model and show lots of examples
- Provide students with content vocabulary prior to teaching a lesson that contains that vocabulary (pre-teaching)
- Tutoring during Delsea One

G & T Modifications

- Ask students' higher level questions that require students to look into causes, experiences, and facts to draw a conclusion or make connections to other areas of learning
- Cross-curricular connections, especially to historical events and people
- Encourage students to explore concepts in depth and encourage independent studies or investigations
- Provide additional rigorous challenge problems for advanced students

At Risk Modifications

- reducing homework length to just those tasks most important for review
- retesting
- speaking to students privately when redirecting behaviors
- tutoring during Delsea One

Formative Assessment

- Exit Ticket
- Group Work
- Guided Practice

- Individual Practice

- Journal Entry

- Kahoot!

- Observation

- Oral Responses

- Poll class to self-analyze their comfort level of the lesson

- Socrative

- Teacher Observation

- Vocabulary Review

Alternate Assessments

Benchmark Assessment

Summative Assessment

- Alternative Assessment

- Marking Period Assessment

- Quiz on Operations on Imaginary Numbers

- Quiz on Radical Equations

- Quiz on Simplifying Radicals

- Unit Test on Complex Numbers

- Unit Test on Operations on Radicals

Resources & Materials

- Chromebook

- Graphing Calculator

- Promethean Board

- Smart Notebook

- Teacher generated worksheets

- Textbooks: Algebra and Trigonometry Structure and Method Book 2 (McDougal Littell), Algebra II Common Core (Pearson)

Technology

- google classroom
- http://kutasoftware.com/
- http://mathxlforschool.com/home_school.htm
- https://create.kahoot.it
- https://njctl.org/
- https://quizizz.com/
- https://socrative.com/
- https://www.desmos.com/
- https://www.resourceaholic.com/
- https://youtu.be/7oWUgtFneco
- https://youtu.be/crNj4-KlWsE
- https://youtu.be/Ef2gOQbDv7M
- https://youtu.be/GP69riiPjLM
- https://youtu.be/n6NBXVUS29U
- https://youtu.be/QsHJStWF-MM
- https://youtu.be/S_OtTdkHiVE
- https://youtu.be/UoB-bPXH7Nc
- Student 1-1 Device (chromebook)
- TI Graphing Calculator

TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
TECH.8.1.12.C.CS4	Contribute to project teams to produce original works or solve problems.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.