Unit 2: Probability, Random Variables, and Probability Distributions

Content Area: **Mathematics**
Course(s): **Traditional Statistics**
Time Period: **November**
Length: **~6-8 weeks**
Status: **Published**

Unit Overview:

Probabilistic reasoning allows statisticians to quantify the likelihood of random events over the long run and to make statistical inferences. Simulations and concrete examples can help students to understand the abstract definitions and calculations of probability. This unit builds on understandings of simulated or empirical data distributions and fundamental principles of probability to represent, interpret, and calculate parameters for theoretical probability distributions for discrete random variables. Interpretations of probabilities and parameters associated with a probability distribution should use appropriate units and relate to the context of the situation.

Essential Questions:

- How can we use the tree diagram to describe chance behavior?
- How can we use the Venn Diagram to model a chance process involving two events?
- How do we calculate and interpret the mean and standard deviation of discrete variables?
- How do we calculate the mean and standard deviation of a binomial random variable?
- How do we calculate the mean and standard deviation of a geometric random variable?
- How do we calculate the probability of a binomial or geometric distributions?
- How do we calculate the probability of an event occurring when using a two-way table?
- How do we describe the effect of transforming a random variable through addition, subtraction, multiplication or dividing by a constant?
- How do we determine if two events are independent?
- How does our simulation model chance behavior and how can we interpret these results?
- How is probability affect when we run long-run relative frequency?
- What are conditional probabilities and how do we calculate them?
- What are the conditions required to solve for a binomial distributions?
- What are the conditions required to solve for a geometric distributions?
- What are the foundational rules to probability?
- What is a binomial distribution?
- What is a geometric distribution?
- What is the addition rule for mutually exclusive events?
- What is the complement rule?
- What is the general addition rule and how can I apply it?

- What is the general multiplication rule and how can I apply it?
- What is the multiplication rule for independent events to compute probabilities?

Enduring Understandings:

- Calculate the probabilities for geometric and binomials random variables and distributions.
- Given that variation may be random or not, conclusions are uncertain.
- Interpret the probabilities for geometric and binomials random variables and distributions.
- Probabilistic reasoning allows us to anticipate patterns in data.
- Probability distributions may be used to model variation in populations.
- Simulation allows us to anticipate patterns in data.
- The likelihood of a random event can be quantified.

Standards/Indicators/Student Learning Objectives (SLOs):

MA.S-IC	Making Inferences and Justifying Conclusions
MA.S-IC.A	Understand and evaluate random processes underlying statistical experiments
MA.S-CP	Conditional Probability and the Rules of Probability
MA.S-CP.A	Understand independence and conditional probability and use them to interpret data
MA.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
MA.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
MA.S-CP.A.3	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
MA.S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
MA.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.S-CP.B	Use the rules of probability to compute probabilities of compound events in a uniform probability model
MA.S-CP.B.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
MA.S-CP.B.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
MA.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = [P(A)] \times [P(B \mid A)] = [P(B)] \times [P(A \mid B)]$, and interpret the answer in terms of the model.
MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.

MA.S-MD	Using Probability to Make Decisions
MA.S-MD.A	Calculate expected values and use them to solve problems
MA.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
MA.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
MA.S-MD.B	Use probability to evaluate outcomes of decisions
MA.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
MA.S-MD.B.5a	Find the expected payoff for a game of chance.
MA.S-MD.B.5b	Evaluate and compare strategies on the basis of expected values.
MA.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Lesson Titles:

- Binomial and Geometric Probability Distributions
- Combining Random Variables
- Conditional Probability and Independence
- Discrete and Continuous Random Variables
- Introduction to Probability
- Tree Diagrams, Combinations and Permutations

Career Readiness, Life Literacies, & Key Skills:

WRK.K-12.P.5	Utilize critical thinking to make sense of problems and persevere in solving them.
WRK.K-12.P.6	Model integrity, ethical leadership and effective management.
WRK.K-12.P.8	Use technology to enhance productivity increase collaboration and communicate effectively.
WRK.K-12.P.9	Work productively in teams while using cultural/global competence.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.

Inter-Disciplinary Connections:

LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
LA.K-12.NJSLSA.R2	Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
LA.K-12.NJSLSA.R4	Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
SOC.6.1.12.EconGE.1.a	Explain how economic ideas and the practices of mercantilism and capitalism conflicted during this time period.
SOC.6.1.12.EconNE.9.a	Explain how economic indicators are used to evaluate the health of the economy (i.e., gross domestic product, the consumer price index, the national debt, and the trade deficit).
SCI.HS-ETS1-4	Use a computer simulation to model the impact of proposed solutions to a complex real-world problem with numerous criteria and constraints on interactions within and between systems relevant to the problem.
SCI.HS-ETS1-3	Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.

Equity Considerations

Amistad Mandate

Topic:

Not Applicable

Materials Used:

Addresses the Following Component of the Mandate:

Holocaust Mandate

Topic:

There is a choice to do one or the both of the items listed below:

- A conditional probability problem about the British founding a Bomber that is lost through enemy action on a mission over occupied Europe.
    - See details here (the problem is on the top of last page)

- Introduce John Kerrich
    - He was a British POW and during his time in captivity he completed statistics problems.
        - The one that would be introduced talks about the Law of Large Numbers with him flipping a coin over a thousand times.
            - See details here

Materials Used:

- A copy of the notes for this unit.

Addresses the Following Component of the Mandate:

- Holocaust Studies

LGBTQ and Disabilities Mandate

Topic (Person and Contribution Addresses):

Alan Turing using Bayes's theorem to help crack the enigma.

Materials Used:

- A copy of the class notes.
- A YouTube video to demonstrate what he did
    - One potential example of this:
        - Click here to watch the video

Addresses the Following Component of the Mandate:

- Bigotry
- Holocaust Studies
- Prejudice
- Social

Climate Change

Not applicable

Asian American Pacific Islander Mandate

Topic (Person and Contribution Addresses):

Susan Ahn Cuddy

- The first Asian American military trailblazer

- The first female gunnery officer
- She eventually worked with the Navy and NSA with codebreaking

Materials Used:

- A copy of the notes
- I will show them this website: Click here to see the website

Addresses the Following Component of the Mandate:

- Social

Instructional Strategies, Learning Activities, and Levels of Blooms/DOK:

For instructional strategies and learning activities please see formative and summative assessment.

Below is the levels of Blooms/DOK:

- Analyze fellow peers work and provide feedback on their strength and room to grow.
- Calculate what is the expected value (mean) and standard deviation for the probability distribution.
- Compare and contrast binomial and geometric distributions.
- Define what is a marginal distribution and apply its formula to various examples.
- Define what is conditional probability.
- Define what is independence in probability.
- Formulate what type of probability problem is present and how to go about providing your conclusions.
- Interpret what does your probability mean in the context of your problem.
- Provide mathematical justification for the steps taken to find the probability
- Recall what is probability.
- Relate your knowledge about probability and probability distributions to various real-world examples like sports, society and more.
- Represent the probability problems using a statistical diagram like the Venn Diagram, Normal curve or density curve.

Modifications

ELL Modifications:

- Continue practicing vocabulary
- Focus on domain specific vocabulary and keywords

- Provide study guides prior to tests
- Read directions to the student
- Read test passages aloud (for comprehension assessment)
- Tutoring during Delsea One
- Vary test formats

IEP & 504 Modifications:

*All teachers of students with special needs must review each student's IEP. Teachers must then select the appropriate modifications and/or accommodations necessary to enable the student to appropriately progress in the general curriculum.

Possible Modifications/Accommodations: (See listed items below):

- Allow for redos/retakes
- Assign fewer problems at one time (e.g., assign only odds or evens)
- Differentiated center-based small group instruction
- Extra time on assessments
- Highlight key directions
- If a manipulative is used during instruction, allow its use on a test
- Opportunities for cooperative partner work
- Provide a word bank
- Provide reteach pages if necessary
- Provide several ways to solve a problem if possible
- Provide visual aids and anchor charts
- Test in alternative site
- Tiered lessons and assignments
- Tutoring during Delsea One
- Use of a graphic organizer
- Use of concrete materials and objects (manipulatives)
- Use of word processor

G&T Modifications:

- Alternate assignments/enrichment assignments
- Encourage Peer Leadership or mentoring
- Extension activities
- Higher-level cooperative learning activities
- Pairing direct instruction with coaching to promote self-directed learning
- Provide additional rigorous challenge problems for advance students
- Provide higher-order questioning and discussion opportunities

- Provide texts at a higher reading level
- Tiered assignments
- Tiered centers

At Risk Modifications

The possible list of modifications/accommodations identified for Special Education students can be utilized for At-Risk students. Teachers should utilize ongoing methods to provide instruction, assess student needs, and utilize modifications specific to the needs of individual students. In addition, the following may be considered:

- Additional help during tutoring/Delsea One
- Additional time for assignments
- Adjusted assignment timelines
- Agenda book and checklists
- Answers to be dictated
- Assistance in maintaining uncluttered space
- Books on tape
- Concrete examples
- Extra visual and verbal cues and prompts
- Follow a routine/schedule
- Graphic organizers
- Have students restate information
- No penalty for spelling errors or sloppy handwriting
- Peer or scribe note-taking
- Personalized examples
- Preferential seating
- Provision of notes or outlines
- Reduction of distractions
- Review of directions
- Review sessions
- Space for movement or breaks
- Support auditory presentations with visuals
- Teach time management skills
- Use of a study carrel
- Use of mnemonics
- Varied reinforcement procedures
- Work in progress check

Formative Assessment:

- Answer Avalanche
- Anticipatory Set
- Closure/Exit-Tickets
- Partner Answer/Analyze Questions
- Polling Questions
- Statistical Activities
- Think-Pair-Share
- Warm-Up

Summative Assessment:

- Benchmark
- Individual Assignment
- Marking Period Assessment
- Performance Task
- Projects
- Quizzes

Resources & Materials:

- AP Sample Questions
- Data investigations
- Each student will participate from either their seats or at one of the boards
- Formula Sheet
- Google Slide lessons
- Independent/Co-operative Learning explorations
- Teacher Generated Worksheets
- TI-84 Calculators
- White board and markers

Technology Materials and Standards

Computer Science and Design Thinking Standards

TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

TECH.8.1.12.A.4	Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.
TECH.8.1.12.A.5	Create a report from a relational database consisting of at least two tables and describe the process, and explain the report results.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.C.CS2	Communicate information and ideas to multiple audiences using a variety of media and formats.
TECH.8.1.12.C.CS3	Develop cultural understanding and global awareness by engaging with learners of other cultures.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.F.CS1	Identify and define authentic problems and significant questions for investigation.
TECH.8.1.12.F.CS2	Plan and manage activities to develop a solution or complete a project.
TECH.8.1.12.F.CS3	Collect and analyze data to identify solutions and/or make informed decisions.